Reading: Carroll & Ostlie sections 5.1-5.4, 8.1-8.1, 9.1-9.5
Exercises [145 pts total]:

(1) **Kinematics of a fast star** [25 pts]
Barnard’s Star is a the highest proper motion star in the sky, with $\mu = 10.4$ “/yr at position angle 356°, and a parallax $\pi = 0.546”$. It has an apparent V magnitude of 9.54. Observing its spectrum, we measure an Hα line position of 655.843 nm (assume this is an “air” wavelength; cf. Table 5.1).

(a) [5 pts] What is the radial velocity of Barnard’s Star?
(b) [5 pts] What is its tangential velocity (i.e., in the plane of the sky)?
(c) [5 pts] What is its total speed relative to the Sun?
(d) [10 pts] Calculate this star’s absolute magnitude, and using Figure 8.16 in Carroll & Ostlie, estimate the spectral type of this star

(2) **Highly Ionized Atoms** [20 pts]
Any atom in which all of the electrons have been removed except one are commonly called “Hydrogen-like ions”, since their electronic transition structure closely matches that of a Hydrogen atom, but with transitions at different energies (we’ll ignore spin-orbit and magnetic coupling).

(a) [10 pt] Following the derivation of the Rydberg constant in Section 5.3, show that the wavelength of light needed to ionize the lowest energy state electron ($n = 1 \to \infty$, the Lyman $\alpha$ transition) of a Hydrogen-like ion scales as $\lambda \sim 1/Z^2$, where $Z$ is the proton number of the atom.

(b) [10 pt] Compute the transition wavelengths of the Hydrogen-like ions He II, C VI, O VIII and Fe XXVI, and state whether they fall into the UV (10-
400 nm), soft X-ray (1-10 nm), hard X-ray (0.01-1 nm) or gamma-ray (<0.01 nm) regimes.

(3) **Magnetic Stars** [15 pts]

Ap stars are a peculiar class of hot star with strong magnetic fields, usually of order 1 T at the photosphere.

(a) [10 pts] Determine the wavelengths of the three components of the Hα line that are produced as a result of Zeeman line splitting (note: the µ in Eqn. 5.22 is the reduced mass of the proton-electron system of a hydrogen atom).

(b) [5 pts] The resolution of a spectrograph is typically quantified as \( R = \frac{\lambda}{\Delta \lambda} \), where \( \Delta \lambda \) is the smallest wavelength range that can be separated at a given wavelength \( \lambda \). What is the minimum spectral resolution needed to resolve the two outer Zeeman lines of Hα in these stars?

(4) **Plotting the HR Diagram** [40 pts]

Using the data from Appendix F and G that is provided on the course website, do the following (it may be helpful to use a spreadsheet/plotting program like Excel or OpenOffice, or a coding language like MatLab, Mathematica or Python):

(a) [10 pts] Using data from Appendix G, plot a “theorist’s” HR diagram, comparing \( \log_{10}(L/L_\odot) \) (y-axis) against \( \log_{10}(T_{\text{eff}}) \) (x-axis) for dwarfs, giants and supergiants. Remember temperature is plotted from right to left! Label each “o” type in each sequence (i.e., M0, K0, etc.) and label the Sun.

(b) [5 pts] On this same plot, indicate lines of constant radius using the Stefan-Boltzmann equation. What are the typical radii of giant and supergiant stars based on your calculations?

(c) [10 pts] Again using data from Appendix G, plot an “observer’s” HR diagram for the same spectral types, comparing M_V (y-axis) against B-V (x-axis). Again, label the “o” spectral types and the Sun.

(d) [10 pts] Describe how these two plots differ. What are some of the possible reasons behind these differences? Pay particular attention to
the shape of the dwarf, giant and supergiant sequences.

(e) [5 pts] On the observer's plot, add in the data for the nearest stars from Appendix F (plot these as a different symbol). In general, what type of stars are the nearest stars? Are there more cool/faint stars or more hot/bright stars in the vicinity around the Sun?

(5) Working with Planck’s Law [25 pts]

Planck’s Law describes the intensity (energy per time per area per frequency per angular area) of radiation from a homogeneous, isothermal source:

\[
\frac{dE}{dtdAd\nu d\Omega} \equiv B_{\nu}(T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1}
\]

Integrating this relation over a spherical hemisphere yields the flux density of blackbody radiation from a surface:

\[
\frac{dE}{dtdAd\nu} = \pi B_{\nu}(T)
\]

(a) [10 pts] Integrate the above relation over all frequencies \((0 < \nu < \infty)\) to derive the total energy flux from a blackbody (the Stefan-Boltzmann Law):

\[
\frac{dE}{dtdA} = \sigma T^4
\]

and show that \(\sigma\), the Stefan-Boltzmann constant, is equal to

\[
\sigma = \frac{2\pi^5 k^4}{15c^2 \hbar^3}
\]

You will need to use the integral:

\[
\int_{0}^{\infty} \frac{x^3 dx}{e^x - 1} = \frac{\pi^4}{15}
\]

(b) [5 pts] Show that the number density of photons of a given frequency \(\nu\) is:
Wein’s Displacement Law states that the peak wavelength of a blackbody distribution scales with the temperature:

$$\lambda T = 2898 \, \mu\text{m-K}$$

Derive this relation, starting from the wavelength form of Planck’s Law:

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$

Note: you will have to solve a transcendental equation, which can be done with a quick bit of coding. Be sure to show how you solved this, otherwise I will assume you just looked up the number (no credit for that!)

(6) **Spectral types, density and temperature** [10 pts]

A 0.5 M\(_\odot\) main-sequence star and a 1.2 M\(_\odot\) giant star can have the same spectral type (M0) but have different atmospheric temperatures. Using an estimate of the mean density of these two stars as a proxy for the atmospheric density (see Appendix G for radii), argue why this would be the case based on the Saha equation.

(7) **Spectral Mnemonics** [10 pts + prize!]

As discussed in class, the spectral sequence of stars is ordered OBAFGKMLTY, with O-type stars being the hottest and most luminous, and Y-type brown dwarfs being the (recently discovered!) coldest and least luminous. There are many mnemonics that are used to remember this seemingly random string of letters, such as

- Oh Be A Fine Girl/Guy Kiss Me, Less Tongue, Yuck!
- Only Boring Astronomers Feel Giddy Knowing Mnemonics Like These (Yawn)
- Our Buddy Adam Feels Great Knowing Maui Life Tops Yours
Obama’s Bailout A Federal Government Killer, Much Luck To You
A list of mnemonics created by UCSD Physics students is linked at
Your assignment: come up with a new mnemonic. There will be a prize
(and international accolades) for the best mnemonic!