Physics 160: Stellar Astrophysics (Fall 2013)

Homework #3

Due Friday October 18th at 5pm
Drop off in the box outside of SERF 340

Reading: Carroll & Ostlie sections 5.1-5.4, 8.1-8.1, 9.1-9.5

Exercises [130 pts total]:

(1) **Hydrogen and the Saha Equation** [40 pts]

Consider two stars: the 5,800 K G-type star the Sun, and the 33,000 K O-type star λ Orionis. Assume that the atmospheres of both stars are pure H and have a density of $10^{-6}$ kg/m$^3$.

(a) [15 pts] For both stars, what is the ratio of neutral H atoms in the ground, first and second excited states ($n = 1, 2$ and $3$)?

(b) [15 pts] For both stars, what fraction of H atoms are ionized? Note that the electron density used in the Saha equation is equal to the density of ionized H atoms for a pure H gas.

(c) [10 pts] H I Balmer α absorption comes from the $n = 2 \rightarrow 3$ transition; H I Lyman α absorption comes from the $n = 1 \rightarrow 2$ transition. How does the line ratio (relative strength) of these two lines differ between the Sun and λ Orionis? Does this depend on the ionization state of H? In which source would both of these lines be extremely weak?

(2) **Looking through liquid water** [25 pts]

The figure below shows the absorption coefficient of pure liquid water as a function of wavelength from Zender & Talamantes (2006, JQRST, 98, 122) is shown above. [Note: the lowest y value should be 0.0001 m$^2$/kg]
(a) [5 pts] Based on the spectral properties of water absorption, explain why the clear ocean appears blue.

(b) [10 pt] If we dive deep enough underwater, it becomes dark. At what point would we see no sunlight whatsoever?

(c) [10 pt] Oceans, lakes and rivers are rarely clear as silt and other material is in suspension in them. Assume a suspension of clay, which has a mass density of 3 g/cm³, with a volume fraction of 10 ppm (parts per million). If you can see your hand only 1 foot into the water, what is the (visible) absorption coefficient of clay?

(3) **Looking through gaseous water** [25 pts]

(a) [10 pt] Water vapor in the air can also absorb starlight, although in this case its density is not constant with height. Show that atmospheric pressure scales vertically as

\[
P(z) = P(0)e^{-z/H}
\]

where

\[
H = \frac{kT}{g\mu m_H}
\]

is the scale height. Compute this height assuming standard temperature and pressure (STP): T = 0 °C = 273 K and P = 100 kPa = 1 bar; and an atmosphere composed (by volume) of 77% N₂, 21% O₂, 1% H₂O and 1% Ar.
(b) [15 pt] Determine the decrease in intensity of visible (\( \lambda = 0.5 \, \mu\text{m} \)) and infrared (1 \( \mu\text{m} \)) light due to water vapor absorption in the atmosphere, as both a fraction and in magnitudes, over 3 scaleheights (95\% of the atmosphere). Assume that the-opacity of water vapor is similar to that of liquid water, and that the surface relative humidity is 50\%, which translates into a surface partial pressure of water vapor of 0.3 kPa.

To make this problem analytically solvable, you need to assume a constant temperature through the atmosphere, which is a terrible assumption. For +5 bonus points, use numerical integration to determine the intensity decline for both wavelengths assuming a temperature decrease with height of -6 K/km.

(4) Random walk [20 pts]

As discussed in section 9.3, the path a light ray takes to escape an optically thick medium can be described as a random walk due to absorption and scattering, with steps equal to the mean free path, L.

In this question, you are going to write a program that performs a series of random walks. Each walk should be 1000 steps of unit length, with the direction being random at each step. You are to compute 1000 such walks, and plot the distribution of final distances from the origin, as well as compute the mean, median and standard deviation of final distances.

Points depend on the dimensionality of your solution: 10 pts for 1D walk, 5 more pts for 2D walk, 5 more pts for 3D walk (so 15 pts for all three sets of walks). If you do more than one set of walks, comment on how the dimensionality of the walk changes the distribution and statistics of the final distances.

Be sure to include both your code (in whatever language) and a figure showing the distribution(s) as part of your solution.