Lecture 16
Star Evolution II: Post Main Sequence Evolution & Pulsating Stars
21 November 2013
Announcements

• HW #7 now online, due Friday
• Remote observing lab is ON for tonight: 7:30pm-9pm in SERF 376
• Becoming Galactic....
ARThUR C CLARKE CENTER FOR HUMAN
IMAGINATION
PRESENTS
BECOMING GALACTIC:
CITIZENS OF THE GALAXY
DECEMBER 3, 2013 5:30 TO 7:30 PM
AT UC SAN DIEGO’S ATKINSON HALL AUDITORIUM

CLARKE CENTER VISITING SCHOLAR
JON LOMBERG

Light refreshments served from 5:30 to 6:00
Free to the public
RSVP info@imagination.ucsd.edu

We are grateful to the Change Happens Foundation for sponsoring this event.

UC San Diego
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The word cosmopolitan literally means a citizen of the Cosmos. Our generation is emerging into the Milky Way, and becoming a planetary species. Jon Lomberg shows some ways that an artist can respond to this amazing fact, combining physics and artistry to create the "cosmic perspective" advocated by Lomberg’s friend and long-time collaborator, Carl Sagan. He will feature a description of his unique Galaxy Garden.

Jon Lomberg is one of the foremost artists inspired by astronomy. His 25 year partnership with Carl Sagan produced some of the most important science inspired art of our time, including EMMY Award winning work as Chief Artist for the original TV series COSMOS. He was Design Director of NASA's legendary Voyager Interstellar Record, humanity's self-portrait launched on a billion year voyage to the stars.
Lecture 16: Star Evolution II:
Post Main Sequence Evolution
21 November 2013

PRELIMS
• Announcements [5 min]

MATERIAL [70 min]
• [5 min] Review
• [30 min] Schonberg-Chandresekhar limit
• [25 min] Post-MS evolution
• [15 min] Pulsating stars

DEMONSTRATIONS/EXERCISES

MATERIALS
Review
• main sequence lifetime of star = for Sun = $10^{10}$ yr, for other stars scales (roughly) as $(M/M_{\text{sun}})^{-2.5}$ (scaling argument)
• how far along does H burning go? This is a Jean’s mass argument

Evolution along the MS
• H->He => $\mu \rightarrow \frac{1}{2} \rightarrow \frac{4}{3}$
• $P \sim \rho T/\mu$ => core $\rho$ & T increase
• $L \sim \epsilon_{\text{nuc}} \sim \rho T^4$ => Luminosity increases too
• Increased energy flow => radius increases
• For Sun, $\Delta L \sim 50\%$, $\Delta T \sim 100$ K since ZAMS
• Faint young Sun paradox: why wasn’t Earth and Mars frozen? (A: internal heating probably played a greater role)

Evolving off the MS
• He core forms – inert and isothermal ($dT/dr \sim$ net energy flux = 0)
• How stable is this? Compute a Jean’s mass!
  o Virial equation with added factor – external pressure term
  o Solve for maximum possible pressure $\Rightarrow$ maximum core radius $\Rightarrow$ maximum core mass $\approx 0.54 \times M_{\text{total}} \times (\mu_e/\mu_{\text{core}})^2$ (Schönberg-Chandresekhar radius)
  o Works out to be 13% $M_{\text{sun}}$ for Sun

Post MS evolution
• Trajectories in HR diagram and primary stages
  o Main sequence turnoff – shell burning, R increases
  o Subgiant branch – He core collapses but doesn’t fuse
  o Red giant phase – envelope cooling to convective
  o Helium flash
  o Horizontal branch
- Asymptotic giant branch & thermal pulsation
- Post-AGB – planetary nebulae & white dwarfs (for low mass stars)

Pulsating stars
- Observations, types
- Driven by instability in He core and H shell burning
- Heat engine model
- Opacity driven pulsation – partial ionization zone
- Instability strips
- Pulsation period => period luminosity relation => Cepheids
Mass limit of the core: Schönberg-Chandrasekhar limit

For a star in isolation, virial theorem is \( 2K + U = 0 \)

\[\begin{align*}
\uparrow & \quad \uparrow \\
\text{kinetic} & \quad \text{gravitational} \\
\text{potential} & \quad \text{energy}
\end{align*}\]

the core of a star is not in isolation - it has an external pressure applied. The correction to the virial theorem is

\[2K + U = 4\pi \rho R_c^3 \rho_s\]

\(K = \frac{3}{2} N K T = \frac{3}{2} \frac{\text{Me}}{\mu \text{MMH}} K T\)

\(U = -\frac{3}{5} \frac{G \text{Me}^2}{R_c}\) (uniform sphere - rough approximation)

\[\Rightarrow \quad \rho_s = \frac{1}{4\pi R_c^3} \left( \frac{3 \text{Me} K T}{\mu \text{MMH}} - \frac{3}{5} \frac{G \text{Me}^2}{R_c} \right)\]

\(\frac{\partial \rho_s}{\partial \text{Me}} = 0 \Rightarrow \frac{K T}{\mu \text{MMH}} = \frac{2}{5} \frac{G \text{Me}^2}{R_c}\)

\[\Rightarrow \quad R_c = \frac{2}{5} \frac{G \text{Me}^2/\mu \text{MMH}}{K T}\]

maximum pressure occurs when \(\frac{\partial \rho_s}{\partial \text{Me}} = 0\)

maximum pressure is (manipulating above)

\[P_{c, \text{max}} = \frac{375}{64 \pi} \frac{1}{G^3 \text{Me}^2} \left( \frac{K T_c}{\mu \text{MMH}} \right)^4\]

below critical mass, adding mass increases \(P \Rightarrow\) can support star
above critical mass, adding mass decreases \(P \Rightarrow\) collapse (gravity wins)
The maximum pressure must support the star

\[ \frac{P_{c,c}}{R} \approx \frac{6M^2}{4\pi R^5} \]  
(hydrostatic equation)

\[ P_{c,c} \approx \frac{6GM^2}{4\pi c^2} \]

Ideal gas in envelope:

\[ P_{e,c} = \frac{P_{c,c} k T_{e,c}}{\mu e M} \]

\[ \approx \frac{3M}{4\pi c^2 R^2} \]

\[ \approx \frac{3}{4\pi} \frac{k T_c}{\mu e M} \frac{M}{R^2} \]

Set these equal:

\[ \frac{GM^2}{4\pi c^2 M} = \frac{3}{4\pi} \frac{k T_c}{\mu e M} \frac{M}{R^2} \]

\[ R_e = \frac{1}{3} \frac{GM^2}{k T_c} \]

\[ P_{e,c} = \frac{81}{81.16} \left( \frac{k T_c}{\mu e M} \right)^4 \]

Set \( P_{e,c} = P_{c,\text{max}} \):

\[ \left( \frac{M_c}{M} \right)^2 = \left( \frac{375}{81.16} \right) \left( \frac{M_c}{M} \right)^4 \]

\[ \frac{M_c}{M} = 0.54 \left( \frac{M_c}{M} \right)^2 \]

Maximum core mass prior to collapse (S-C. produced value & 0.37 more careful derivation)

for \( M_c = 0.5, \mu c = 4/3 \), \( M_c \approx 0.20 \) M\( \odot \) in reality \( M_c \approx 0.30 \) M\( \odot \) due to additional pressure support from degeneracy (esp. low mass stars)

Beyond this, star progresses to next phase of evolution
After main sequence evolution, formation of the core + shell burning, and collapse of the core, the star is well on its way toward stellar death. Here are the various cycles for a low-to-intermediate-mass star (0.6 ≤ M ≤ 8 M☉).

1. At this stage, the core has formed + H is burning in shell outside core. Line in shell to > L red, and shell does not easily expand to accommodate excess energy production. So entire star expands (work done on star). This is the main sequence turn off.

2. When the Schönberg-Chandrasekhar mass limit is breached, core contracts, releasing gravitational potential energy on Kelvin-Helmholtz timescale (~10^7 yr). To compensate for additional energy output (gravitational + contracted + heated shell) star expands with constant L red → T eff decrease — this is the subgiant branch.

3. Eventually envelope cools to point where convection kicks in — this is much more efficient in getting luminosity out, so L increases. In fact, this is almost reverse of pre-main-sequence phase (fully convective contracting star) so star heads up Hayashi track. This is the red giant phase and is accompanied by dredge-up of deep elements leading to modified photospheric abundances.
4. Core has contracted + heated up (virial theorem!) such that triple α can occur - this is the Helium flash. The production of energy causes core to expand and pushes out H-burning shell, which decreases in intensity. As a result, envelope shrinks, and L decreases, Teff decreases.

We now have a He-burning star that proceeds along horizontal branch (note: some of these stars are pulsating).

5. 12C core forms as He is depleted, and density again goes up as Lr → 0 in core. We now have a multi-shell burning star (H + He), which repeats history — envelope expands, star cools at nearly constant L until S-C mass limit is reached for C core.

The star then proceeds up asymptotic giant branch.

6. Thermally pulse AGB stars - at top of AGB track, feedback between shell can cause quasiperiodic bursts of burning activity; e.g.:

- H shell contracts + reignites
- He shell contracts + reignites, expanding H shell + shutting it down
- He shell turns down, H shell contracts + reignites, repeating cycle

These cycles can last 10^3-10^5 yr.

Convection zones in stars w/ M > 2 M_⊙ can dredge up C to photosphere; if X_C/X_0 > 1, then a carbon star is made, which exhibit unusual molecular features (CN, C2, C3) and also show

^{43}Ti, an unstable isotope w/ T_1/2 ~ 2×10^5 yr

AGB stars are also (slowly) mass, up to 10^-7 M⊙/yr (superwind)
post-AGB phase: as winds push material in envelope away, the star becomes optically thin down to remnant core - $T_{eff}$ increases dramatically. The expanding shell of gas eventually becomes a planetary nebula ($\sim 10^4 - 10^5$ yr).

Eventually, the envelope is cleared and remnant core (which is supported by degeneracy pressure and does not fuse anything) cools and dims at constant radius. These stars have masses $<1.4 M_\odot$ and radii $\sim R_{\oplus}$, and may be the C, or O/MgNe "balls" depending on initial mass of star.

Note that the timescales of these phases depend on stellar mass. If one has a casual collection of stars such as a cluster, you can use the pattern of these on the H-R diagram to determine age of cluster.
This is the maximum pressure \( P_c \) a relativistic degenerate can, and ultimately sets the limit as to how massive this can can be.

The gravitational pressure pushing downward is:

\[
P_c = C_1 \frac{G M^2}{R^4}
\]

where \( P_c = C_2 \langle \rho \rangle \), \( \frac{C_2}{4\pi} \frac{M}{R^3} \)

\[\uparrow\text{constant depends on polytrope relation}\]

\[\approx 55\text{ for } n = 3\text{ polytrope}\]

\[\approx 7.5\text{ for } n = 3\text{ polytrope}\]

\[P_{\text{gravitation}} \geq P_{\text{degeneracy}}\text{ where } C_1 \frac{G M^2}{R^4} \geq \frac{(3\pi)^{1/3}}{4} \frac{K_C}{C_1} \left( \frac{7}{4} \frac{C_2}{3\pi} \frac{M}{R^3} \right)^{4/3}\]

\[= \frac{M^{4/3}}{16\pi} \left( \frac{9}{4} \right)^{1/3} \left( \frac{C_2}{C_1} \right)^{4/3} \left( \frac{K_C}{C_1} \right)^{1/3} \left( \frac{7}{4} \frac{M}{R^3} \right)^{4/3}\]

\[= M > \sqrt{\frac{3}{2}} \frac{9}{128\pi} \frac{C_2}{C_1} \left( \frac{K_C}{C_1} \right)^{1/2} \left( \frac{7}{4} \frac{M}{R^3} \right)^{1/2}\]

quantum, relativistic + gravity in core

\[\text{term } = P_{\text{core}}\text{, mass}\]

\[\sqrt{\frac{K_C}{G}} = 2 \times 10^{-8}\text{ kg}\]

For \( \frac{M}{R} = 0.5 \), \( C_1 = 7.5 \), \( C_2 = 55 \) this all reduces to

\[M \geq 1.4 M_\odot\]

while direct mass limit

\[M_{\text{max}} = 1.4 M_\odot\]