Virial theorem

- Center of the sun is 15 MK - Where does all this thermal energy come from?
  - “Burning”
    - chemical bond potential ~ 5 eV
    - take methane CH4 = 20 eV/molecule x 6x10^23 molecules/16 g x 1.6e-19 J/eV x 1000 g/kg = 108 J/kg = ε
    - timescale for burning the entire Sun = εM/L = 108 J/kg x 2 x 10^30 kg / 4 x 10^26 W = 5x10^11 s = 2x10^4 yr – too short!
  - Gravity
    - Origin of stars – ISM [SLIDE]
    - For ρ_{ISM} = 10^{-20} kg/m^3, R(Msun) = R_{\odot} (ρ_{sun}/ρ_{ISM})^{1/3} ≈ 5x10^6 R_{\odot} ≈ 0.1 pc.
    - Gravitational energy release = -Ω = GM^2[1/R_{\odot} – 1/0.1pc] = 4x10^{41} J
    - If entire star absorbed that, NkT = (M/m_H)kT = -Ω => T = Ω m_H/M_k ≈ 2x10^7 K – hot enough!

- Derive virial theorem
  - Show that kinetic energy gives just about the right answer
- Effect of radiating star losing energy => star shrinks and temperature increases (negative heat capacity)
- Derive Kelvin-Helmholtz timescale for sun – 20 Myr
  - Implications for spectral sequence
  - realization that solar system is much older – alternate energy source needed

Nuclear physics

- Primer on the fundamental particles
- Force carrying bosons
Range over which bosons can carry range = Compton wavelength
- pion – exchange of strong force – 140 MeV => d \( \approx 10 \text{ fm} \)
- W,Z bosons – exchange of weak force – [LOOK UP VALUES]
- Photons, gravitons -> \( \infty \)

Addition of Higgs
- Nuclear binding energy – energy required to break a nucleus into free particles
  - Example of \(^4\text{He}:\) BE = 27.4 MeV
  - BE per nucleon – peaks at Fe, sets pathways for nuclear physics
  - BE per mass – nuclear >> gravity
- Stability: energy diagram of nuclei – n & p states
  - What nuclei are stable depends on relative distribution of n & p
  - Excited state might also be unstable
- Decay modes

\textbf{pp chain & tunneling}
- Primary reactions [SLIDE]
  - pp I
  - pp II and pp III side chains
  - CNO chains
  - Net energy release & products (all the same)
- Why doesn’t everything fuse?
  - Potential energy diagram up to pion interaction radius – equivalent T – this is the lowest Coulomb barrier!
  - Tunneling probability – gives rough estimate of T required for fusion
Higher coulomb barrier for higher Z nuclei => higher temperatures required
This is an issue for + + interactions, not with neutrons (but the latter are rare – decay and weak interaction)

Nuclear reaction rates
• Energy generation term (per mass)
  o Derivation of cross-section
  o Gamow peak
  o How this folds into source term in radiative transfer equation
• Reaction rates
  o general form and dependence on abundances
  o reaction chain
Announcements

• HW #4 now posted, due Friday @ 5pm in box outside my office SERF 340
• HW #3 solutions up tonite
• Local lab => tomorrow 7-9pm at Torrey Pines Glider Port *WEATHER PERMITTING*
  – make-up date: Monday 10/28
• Brown Dwarf Talk: Monday 10/28 3-4pm @ Mayer 2623
• NEXT WEEK: Midterm (no homework)
  – equation sheet will be provided
• Astrophysics seminar
Advances in galaxy-formation simulations: calculating mock observables & using a more-accurate numerical technique

Dr. Chris Hayward, Heidelberg Institute of Theoretical Science
What power stars?

Here's some guesses.

"Burning" = chemical energy, e.g., burning gasoline

...take CH₄ = 4 C-H bonds - how much energy released if you just dissociate these bonds?!

...estimates... bond strength ≈ 5 eV (less than ionization)

energy release per mass = 5 eV \( \times \) \( \frac{1.6 \times 10^{-19} \text{ J/eV}}{6.022 \times 10^{23} \text{ molecules}} \) \( \times \) \( \frac{1 \text{ mole}}{16 \text{ g}} \) \( \times \) \( \frac{1000 \text{ J/kg}}{1 \text{ g}} \) = \( 3 \times 10^{-7} \text{ J/g} \)

burn the Sun! = \( (3 \times 10^{-7} \text{ J/g})(2 \times 10^{30} \text{ kg}) = 6 \times 10^{23} \text{ J} \)

current luminosity = \( 4 \times 10^{26} \text{ W} \)

\( \Rightarrow \) lifetime = \( \frac{E}{L} = 1.5 \times 10^{11} \text{ s} \left( \frac{1 \text{ yr}}{\pi \times 10^5} \right) \)

\( \approx 5 \times 10^8 \text{ yr} \) - OK for Bible, but not radiodating of rocks on Earth

"Gravity" - assume star started off really large.

UGWTA \( GM^2 \left( \frac{1}{R_{\text{Sun}}} - \frac{1}{\infty} \right) = \left( \frac{4 - 6 \times 10^{-11} \text{ m}^3/\text{kg}^2}{7 \times 10^8 \text{ m}} \right) \)

\( \approx 4 \times 10^{41} \text{ J} \leq 10^4 \) more than "gas"

lifetime = \( \frac{4 \times 10^{41} \text{ J}}{4 \times 10^{26} \text{ W} \times \frac{1 \text{ yr}}{\pi \times 10^5}} = 3 \times 10^7 \text{ yr} \)

- still 100x too short

"Nuclear option" - 4He \( \rightarrow \) He releases \( 4\overline{\text{He}}^2 - \overline{\text{He}}^2 \)

\( = (4 \times (939.272 \text{ MeV} - 3727.379 \text{ MeV})) = 25.7 \text{ MeV} \)

for the Sun: \( 25.7 \text{ MeV} \left( \frac{1.6 \times 10^{-13} \text{ J/MeV}}{0.7} \right) \left( \frac{2 \times 10^{30} \text{ kg}}{1.7 \times 10^8 \text{ kg}} \right)^{1/4} \)

\( \approx 8 \times 10^{44} \text{ J} \)

\( \Rightarrow \) total = \( \frac{8 \times 10^{44} \text{ J}}{4 \times 10^{26} \text{ W}} \left( \frac{1 \text{ yr}}{\pi \times 10^5} \right) = 10^{11} \text{ yr} \) - now we're talking!
“The Pillars of Creation”
Eagle Nebula (Messier 16)

\[ \approx 7 \text{ light-years} \]
Until the 1930s-1940s, the energy source for the Sun was still remained mysterious. Why was it so hard to power the Sun?

**Virial theorem**

Recall hydrostatic equilibrium:

\[
\frac{dp}{dr} = -\frac{GM(r)\rho(r)}{r^2}
\]

Multiply by \(4\pi r^3\) and integrate from center to star to \(R\):

\[
\int_0^R \frac{dp}{dr} 4\pi r^3 \, dr = \int_0^R 4\pi r^2 M(r)\rho(r) \, dr
\]

\[
\int_0^R 4\pi r^2 p \, dr = \int_0^R \rho(r) 12\pi r^2dr \quad \text{(integration by parts)}
\]

\[
\rho(R) = 0 \quad r(\Theta) = 0
\]

\[
-2 = \int_0^R 3\rho(r) 4\pi r^2 dr
\]

\[
-2 = \int -3 \rho dV
\]

\[
- G \int_0^R \frac{4\pi r^2 \rho(r) M(r)}{dM} \frac{M(r)}{r} = - G \int \frac{\rho dM}{r} \equiv \mathbb{E}_G \quad \text{total gravitational potential energy}
\]

Remember:

\[
p = \frac{2}{3} n \left(\frac{3}{2} u^2\right) = \frac{2}{3} n u \quad \text{average KE/particle}
\]

\[
u = \frac{u}{m} \quad \text{thermal energy density}
\]

\[
\Rightarrow -3 \int \rho dV = -3 \int \frac{2}{3} n u dV = -2 \int u dV = -2 KE_{\text{total}}
\]

\[
\Rightarrow 2 KE_{\text{total}} + \mathbb{E}_G = 0 \quad \text{or} \quad E_{KE} = -\frac{1}{2} E_{grav}
\]

**Virial theorem**
Numerical estimate for Sun:

\[ E_{KE} \approx \frac{3}{2} N k T < T > \times 10^6 \text{ to } 10^7 \text{ K} \]

(all particles \( N = \frac{M_e}{m} \times \frac{2 \times 10^{33} g}{1.7 \times 10^{-44} g} \approx 10^{57} \text{ atoms} \))

\[ = 1.5 \times 10^{57} \times 10^6 \text{ K} \times 1.4 \times 10^{-16} \text{ erg/ K} \approx 2 \times 10^{48} \text{ erg} \]

\[ \Omega G \approx -\frac{G M_e^2}{R_6} = \frac{\left(6.7 \times 10^{-6} \text{ cm}^3/\text{g}^2\right) \left(2 \times 10^{33} \text{ g}\right)^2}{\left(7 \times 10^{10} \text{ cm}\right)} \]

\[ = 4 \times 10^{48} \text{ erg} \]

What is the consequence of virial theorem for a radiating star?

\[ F_{total} = E_{KE} + E_{grav} = \frac{1}{2} \Omega G \]

 radiation \( \Rightarrow \) \( \Delta E < 0 \) \( \Rightarrow \) \( \frac{1}{2} \Delta \Omega G < 0 \)

\[ \Omega G = -\frac{k G M^2}{R^6} \quad (k = \frac{3}{5} \text{ for uniform sphere}) \]

\( \Delta \Omega G < 0 \) \( \Rightarrow \) \( R \) must decrease

but \( E_{KE} = -\frac{1}{2} \Omega G \) \( \Rightarrow \) \( \Delta E_{KE} = -\frac{1}{2} \Delta \Omega G > 0 \)

\( \Rightarrow \) temperature increases

\[ \frac{1}{2} \text{ energy goes into radiation} \]

\[ \frac{1}{2} \text{ energy goes into heating core} \]

\( \Rightarrow \) this is Kelvin-Helmholtz contraction

What is the timescale?

\[ \frac{dE}{dt} = -\text{L} = -\frac{1}{2} \frac{d\Omega G}{dt} = \frac{k G M^2}{2} \frac{dr}{dt} \]

\( \Rightarrow \) \( \frac{dr}{dt} = \frac{k G M^2}{2 L} \frac{1}{r^2} \)

\( \Rightarrow \) \( \tau_{KH} = \frac{k G M^2}{2} \frac{1}{L} \) (\( \frac{1}{R_{final}} - \frac{1}{R_{initial}} \))

if \( R_{initial} \approx R_{final} \)

\( \tau_{KH} \approx \frac{k G M^2}{2} \frac{1}{L} \)

\( \tau_{sun} = \frac{1}{2} \Omega G \]

\( \tau \) for sun, this is 20 Myr!
A Little Primer on Nuclear Physics

Fundamental Particles:

- Leptons: e⁻, µ⁻, τ⁻, νₑ, νµ, ντ
- Quarks: u, d, s, c, t, b

- Force Bosons:
  - γ (Photon) (EM)
  - g (Gluon) (Strong)
  - W⁺, ± (Weak)
  - Z (Weak)
  - H (Graviton) (Gravity)

Protons and neutrons are composites of u and d quarks.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Spin</th>
<th>Charge</th>
<th>Baryon</th>
<th>$m c^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>1/2</td>
<td>+2/3</td>
<td>1/3</td>
<td>2.4 MeV</td>
</tr>
<tr>
<td>d</td>
<td>1/2</td>
<td>+1/3</td>
<td>1/3</td>
<td>4.8 MeV</td>
</tr>
<tr>
<td>p = uud</td>
<td>1</td>
<td>+1</td>
<td>1</td>
<td>938.28 MeV</td>
</tr>
<tr>
<td>n = udd</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>939.57 MeV</td>
</tr>
</tbody>
</table>

Note: $M_n > M_p$

Force Carrying Bosons:

- Odd integer spin (1, 3, ...): These particles repel $→$ $X, g, W, \gamma$
- Even integer spin (0, 2, ...): These particles attract $→$ graviton

Force conveyed by exchange of virtual boson satisfying:

$$\Delta E \Delta t < \hbar \Rightarrow \Delta d = \Delta t c < \frac{h c}{\Delta E}$$

- Photon: $\Delta d < \lambda$
- Massive Boson: $\Delta d < \frac{h c}{m c^2}$ (Compton d)

Pauli exclusion
### Quarks

- **$u$** (up)
- **$c$** (charm)
- **$t$** (top)
- **$d$** (down)
- **$s$** (strange)
- **$b$** (bottom)

**Charge assignments:**
- $q = +2/3$
- $q = -1/3$
- $q = 0$
- $q = -1$

### Leptons

- **$e_e$** (electron neutrino)
- **$\nu_\mu$** (muon neutrino)
- **$\nu_\tau$** (tau neutrino)
- **$e$** (electron)
- **$\mu$** (muon)
- **$\tau$** (tau)

**Charge assignments:**
- $q = 0$
- $q = -1$

### Bosons

- **$\gamma$** (photon)
- **$Z$** (Z boson)
- **$W$** (W boson)
- **$g$** (gluon)
- **$Higgs$** (boson)

**Graviton?**

Source: AAAS

A more relevant number is the nuclear binding energy B.E. which describes the energy required to break a nucleus up into its constituent pieces.

\[ \text{B.E.} = \sum m_i c^2 - Mn c^2 \]

\[ = \left[ (A-2)Mn + 2M_p + Mn \right] c^2 \]

Consider the \( \frac{\text{4}}{2} \) He nucleus:

\[ \text{B.E.} = 2M_n c^2 + 2M_p c^2 - Mn c^2 \]

\[ = 2(939.57274 \text{ MeV}) + 2(938.2794 \text{ MeV}) - 3726.3500 \text{ MeV} \]

\[ = 27.4 \text{ MeV} \]

![Image description: A comparable estimate can be made by assuming nucleons form a degenerate material with density \( N = A/4 \pi R^3 \)

with \( R = 1.4 \times 10^{-13} \text{ A}^\frac{1}{3} \) cm

\( \Rightarrow N = 9 \times 10^{37} \text{ cm}^{-3} \Rightarrow \rho \approx 1.5 \times 10^{14} \text{ g cm}^{-3} \)

The Fermi momentum \( \text{p_F} = \frac{\hbar c}{\pi} \left( \frac{2}{\rho} \right) \]

\[ = \frac{1.24 \times 10^{-8} \text{ ev cm}}{0.007} \frac{2}{9 \times 10^{37} \text{ cm}^{-3}} \]

\[ = 2.7 \times 10^8 \text{ ev} \]

The Fermi energy \( \text{E_F} = \frac{p_F^2}{2m} = \frac{(\text{p_F})^2}{2mc^2} \]

\[ = \frac{(2.7 \times 10^8 \text{ ev})^2}{2 \times 9.4 \times 10^6 \text{ ev}} \]

\[ = 40 \text{ MeV - close!} \]
Note that for $^{4}\text{He}$, the binding energy per nucleon is:

$$f = \frac{BE}{A} = 6.5 \text{ MeV}$$

In fact, $f$ rises from $\text{H} \rightarrow \text{Fe} (x \approx 0.5 \text{ MeV})$ then declines, thus setting the pathways for energy generation via nucleosynthesis.

We can also measure this as binding energy per mass:

$$\frac{BE}{4m_n} = \frac{BE}{m_n c^2} \approx \frac{7 \text{ MeV}}{4 \times 10^{-3} \text{ MeV} c^2} \approx 1.7 \times 10^{18} \text{ erg/g}$$

Compare to gravity:

$$\frac{BE}{\mu m} = \frac{GM^2}{R \cdot M} \approx 2 \times 10^{15} \text{ erg/g} \text{ for Sun}$$

If fusion is such an efficient energy source, and H plentiful in the Universe, why doesn't everything fuse? A: Coulomb barrier of $2.72 \text{ MeV}$ is powerful barrier; thermal equivalent is:

$$T = \frac{2.72 \text{ MeV}}{k_B} \approx 2.73 \times 10^{10} \text{ K}$$

For a Maxwell-Boltzmann distribution at $T = 10^7 \text{ K}$, the factor of protons is:

$$\int_{10^{10} \text{ K}}^{\infty} \frac{dn(c)}{n}$$
Stability of Nuclei

\[ M_{\text{nuc}} > M_{\text{p}} \Rightarrow \text{energetic motivation for nuclear decay} \]

\[ \begin{array}{c}
\text{u} \\
\text{d} \\
\text{u} \\
\text{d} \\
\text{p}
\end{array} \]

\[ \frac{1}{\sqrt{2}} (\text{u}\bar{u} - \text{d}\bar{d}) \]

\[ +1 \quad \uparrow \cdot 0 \]

\( \beta \) decay = decay of free neutron

\[ n \rightarrow p + e^- + v^- \]

\[ \text{Chase lifetime } T \approx 10 \text{ minutes! } \Delta E = 782 \text{ keV} \]

**Question**: Since neutrons are present in many atoms, why aren't atoms inherently unstable?

Nuclei held together by **strong nuclear force** through exchange of pions or mesons.

<table>
<thead>
<tr>
<th>Particle</th>
<th>Quark Configuration</th>
<th>Charge</th>
<th>Spin</th>
<th>(Mc^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (\pi^0)</td>
<td>(\frac{1}{\sqrt{2}} (\text{u}\bar{u} - \text{d}\bar{d}))</td>
<td>0</td>
<td>(\uparrow \cdot 0)</td>
<td>134.98 MeV</td>
</tr>
<tr>
<td>1 (\pi^0)</td>
<td>(\bar{u}d)</td>
<td>+1</td>
<td>(\uparrow \cdot 0)</td>
<td>139.57 MeV</td>
</tr>
<tr>
<td>-1 (\pi^0)</td>
<td>(\bar{d}u)</td>
<td>-1</td>
<td>(\uparrow \cdot 0)</td>
<td>139.57 MeV</td>
</tr>
</tbody>
</table>

\(\pi\) meson sets size scale of nuclei:

\[ \Delta d < \frac{\hbar c}{m_{\pi} c^2} = \frac{1.2 \times 10^{-6} \text{eV}\cdot \text{m}}{140 \times 10^6 \text{eV}} \leq 10^{-14} \text{m} \approx 100 \text{ fm} \]

Largest stable nucleus = \(^{208}\text{Pb}\)

\[ R_{\text{nucleus}} \approx r_0 A^{1/3} = 1.25 (208)^{1/3} \approx 7 \text{ fm} \]

\(\Pi\) meson exchange couples p+n in nucleus within \(\approx 100 \text{ fm}\) box.
Consider energy diagram for nucleus

1. Energies of n, p ≈ rest mass energies
2. \( m_n > m_p \) ⇒ neutron energy state higher
3. \( n \) and \( p \) are distinguishable particles ⇒ separate energy states
4. Pauli exclusion: \( 2n \) or \( 2p \) cannot occupy same state

Free neutron decay energetically allowed

What about \( \text{H}_2 = np \)?

\( n \) cannot go to \( p \) because that would overfill \( p \) state (Pauli exclusion)
Only available states have higher \( E \)
⇒ not favored

\( \text{H}_2 \) is stable

\( \text{H}_3 \) is unstable because \( 2^{12}n \) can switch to lower \( p \) state

\( \text{H}_3 \rightarrow 2^{12}\text{He}^3 + e^0 + \nu^0 \)

\( T_{1/2} = 12.3 \text{ yr} \), \( \Delta E = 18 \text{ keV} \)

\( \text{C}^{12} \) is stable because electrostatic repulsion

moves proton energy states apart;

again \( n \) can't decay to \( p \) as it would require energy

In general \( Z \approx A - T \)
\[ \neq_p + n \]
$^7\text{N}^{13}$ is unstable because last $\beta$ has greater energy than open n state $\Rightarrow$ reverse $\beta$ decay

$^7\text{N}^{13} \rightarrow 6\text{C}^{13} + \beta^- + e^0 + 0.78\text{MeV}$

$T_\frac{1}{2} \approx 10\text{min}$

$\Delta E = 2.22\text{MeV}$

$^7\text{N}^{13}$ is used in PET imaging, and part of CNO cycle in stars

Other decay modes

Proton capture: proton incorporated into nucleus; quantum tunneling process

$e^- + 8\text{O}^{16} + 1\text{H}^1 \rightarrow 9\text{F}^{17} + \gamma \text{ EM interaction}$

Alpha decay: split of nucleus that results in ejection of $^4\text{He}$ atom (very stable)

$e.g., \text{H}^1 + 7\text{N}^{15} \rightarrow 6\text{C}^{12} + 2\text{He}^4$ (CNO cycle)

Proton-neutron emission: quantum tunneling out of nucleus - occurs in excited states of massive nuclei (also double decay)
Consider step (1)

\[
\text{Mass (left)} = 2m_p c^2 = 1.836 \text{ GeV} \\
\text{Mass (right)} = M_{\text{He}}^2 c^2 + M_{\text{e}}^2 c^2 \leq M_{\text{He}}^2 c^2 + M_{\text{e}}^2 c^2 = 1.879 \text{ GeV} \\
\Delta m c^2 = 1.8 \text{ MeV} \leq \text{does not seem to work!}
\]

However, \( M_{\text{He}}^2 = 1876 \text{ MeV} - 2.2 \text{ MeV} \text{ in binding energy, } M_{\text{He}}^2 \text{mp} \)

\[ \Rightarrow \text{Mass (left)} > \text{Mass (right)} \Rightarrow \text{reaction occurs} \]

Energy released \(= 2.2 \text{ MeV} - 1.8 \text{ MeV} = 0.42 \text{ MeV} \)

\( \Delta \text{m c}^2 \)

**Bonus**: \( \bar{\text{e}}^0 \) will immediately annihilate with \( e^0 \rightarrow 2 \times 0.511 \text{ MeV} = 1.02 \text{ MeV} \)

\[ \Rightarrow Q_{\text{total}} = 1.44 \text{ MeV} \]

What about neutrinos? This is a weak interaction

**Neutrinos have energy spectrum:**
- released: take away some of reaction energy (lost by neutrino radiation)

\[ <E_{\nu}> = \int E_{\nu} n(E_{\nu}) \text{d}E_{\nu} = 0.26 \text{ MeV} \]

\[ \int n(E_{\nu}) \text{d}E_{\nu} \]

\[ \Rightarrow \text{net reaction} = 1.18 \text{ MeV} \]
Step 2: 8 released $\Rightarrow$ this is an EM reaction $\Rightarrow$ fast rxn

Step 3: combining 7 + 2 particle $\Rightarrow$ 4x colored barrier
   $\Rightarrow$ slow rxn

These steps must be
Steps 1 + 2 must be done twice for step 3 $\Rightarrow$ net rxn is:

$$6 \, ^1H + 2 \, ^1H \rightarrow 2 \, ^2H + 2 \, ^4He + 2 \, ^0\alpha + 2 \, ^0\nu$$

or

$$6 \, ^1H + 2 \, ^4He + 2 \, ^0\nu \rightarrow 2 \, ^2H + 2 \, ^4He + 2 \, ^0\nu$$

Net energy released $\rightarrow$ U loss

$$= 2 \times 1.44 + 2 \times 5.49 + 12.85 - 2 \times 0.26 = 26.2 \text{ MeV}$$

There are in fact a chain of H fusion rxns:

- $^1H + ^1H \rightarrow ^2H + ^0\alpha + ^0\nu \rightarrow \text{reverse } \beta \text{ decay}$
- $^1H + ^4He \rightarrow ^2He + ^3H + ^0\nu \rightarrow \alpha \text{ capture}$

- $^2He + ^2He \rightarrow ^2He + ^2He + ^1H + ^1H$

- $^3H + ^1H \rightarrow ^2He + ^2H + ^0\nu \rightarrow \text{proton capture}$

- $^4Be^+ + ^0\nu \rightarrow ^3Li^+ + ^0\nu$ (99.7%) $\rightarrow^0.35\%$

- $^3Li^+ + ^1H \rightarrow ^4Be^+ + ^1H \rightarrow^4Be^+ + ^1H \rightarrow ^5B^++^1H$

- $^5B^+ \rightarrow ^4Be^+ + ^0\nu + ^0\nu$

Note: this U was detected at rate too low to solve neutrino problem
$^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e^+ + \nu_e$

$^2\text{H} + ^1\text{H} \rightarrow ^3\text{He} + \gamma$

$^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + 2^1\text{H}$

$^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma$

$^7\text{Be} + e^- \rightarrow ^7\text{Li} + \nu_e$

$^7\text{Li} + ^1\text{H} \rightarrow 2^4\text{He}$

$^7\text{Be} + ^1\text{H} \rightarrow ^8\text{B} + \gamma$

$^8\text{B} \rightarrow ^8\text{Be} + e^+ + \nu_e$

$^8\text{Be} \rightarrow 2^4\text{He}$

PP chains for H fusion

http://burro.cwru.edu/Academics/Astr221/StarPhys/ppchain.gif
The $^4\text{He}$ can also be produced through cycles using higher elements, e.g., CNO cycle:

1. $^1\text{H} + 6 \, ^{12}\text{C} \rightarrow 7 \, ^{13}\text{N} + \gamma \quad (1.85 \text{ MeV})$

2. $7 \, ^{13}\text{N} \rightarrow 6 \, ^{12}\text{C} + ^{1}\text{H} + 0\nu$

3. $^1\text{H} + 6 \, ^{13}\text{C} \rightarrow 7 \, ^{14}\text{N} + \gamma \quad (7.54)$

4. $^1\text{H} + 7 \, ^{14}\text{N} \rightarrow 8 \, ^{15}\text{O} + \gamma \quad (2.35)$

5. $8 \, ^{15}\text{O} \rightarrow 7 \, ^{15}\text{N} + ^{1}\text{H} + 0\nu$

6. $^1\text{H} + 7 \, ^{15}\text{N} \rightarrow 6 \, ^{12}\text{C} + 2 \, ^4\text{He}$

Net: $4 \, ^1\text{H} \rightarrow 2 \, ^4\text{He} + 2 \, ^{1}\text{H} + 2 \, 0\nu + 3\gamma$

\[ 26.73 \quad 1.70 \]

(25.03 net)

\[ \uparrow \]

slightly less - two escaped $\nu$'s

CNO cycle uses $^{12}\text{C}$ as catalyst, although side reactions cause slow conversion $^4\text{He} \rightarrow ^{12}\text{C}$.