Physics 223: Stellar Astrophysics
Homework #1 (updated 10/4)
Due Friday October 7th at 5pm in box in front of SERF 340

**Reading:** HKT Chapters 1 & 2

**Exercises:** [115 pts total]

(1) **Mean molecular weights [25 pts]**

Compute the mean molecular weights of the following gases; for relative abundances of elements, you can consult either Table 1 of Asplund et al. (2009): [http://adsabs.harvard.edu/abs/2009ARA%26A..47..481A](http://adsabs.harvard.edu/abs/2009ARA%26A..47..481A) or the abundances.xlsx table included with this assignment.

(a) [15 pts] Use the abundances.xlsx spreadsheet to compute $\mu_I$, $\mu_e$, $\mu_{\text{total}}$, $X$, $Y$ and $Z$ for the seven abundance patterns provided. For the total mean molecular weight, assume a fully ionized ($y = 1$) state (both ions and electrons contribute). It is recommended that you use a spreadsheet or code to compute these.

(b) [10 pts] Compare the total mean molecular weights for two purely ionized gases: one with Solar abundances (from part a) and one for a pure carbon white dwarf. Explain how the difference in these values relate to the relative sizes of H-rich stars and C-rich white dwarfs, using the equation of hydrostatic equilibrium as a guide.

(2) **Spectrophotometric Distance Estimates of Binary Stars [20 pts]**

It is common to estimate the distance of a star based on its spectral type and the average absolute magnitude for that spectral type (determined for similar stars with distance measurements). However, if the star is a binary, this distance will be underestimated.

(a) [5 pts] Proxima Centauri is the closest star to the Sun. Based on its measured apparent magnitude and parallax (find these on SIMBAD), compute the absolute magnitude of this star in B, V and J bands, and compare to the absolute magnitudes of the Sun in these bands.

(b) [5 pts] Star 42 Makeyupus has an identical spectrum and color as Proxima Centauri but an apparent J magnitude of 12.45. How far away
do you estimate this star to be?

(c) [10 pt] It is discovered that 42 Makeyupus has a binary companion that is 30% as bright as the primary. Assuming the primary is identical to Proxima Centauri, re-estimate the distance of this system based on the brightness of the primary alone (be careful with your definition of magnitude here!)

(3) Plotting the HR Diagram [30 pts]

Using the data provided on the course website, do the following (it may be helpful to use a spreadsheet/plotting program like Excel or OpenOffice, or a coding language like MatLab, Mathematica or Python):

(a) [10 pts] Using the file HR-theory.xlsx, plot a “theorist’s” HR diagram, comparing $\log_{10}(L/L_\odot)$ (y-axis) against $\log_{10}(T_{\text{eff}})$ (x-axis) for dwarfs, giants and supergiants. Remember temperature is plotted from right to left! Label each “o” type in each sequence (i.e., M0, K0, etc.) and label the Sun.

(b) [5 pts] On this same plot, indicate lines of constant radius using the Stefan-Boltzmann equation. What are the typical radii of giant and supergiant stars based on your calculations?

(c) [5 pts OPTIONAL] Using the NearestStars.xlsx and BrightestStars.xlsx files, plot an “observer’s” HR diagram for the same spectral types, comparing $M_V$ (y-axis) against $B-V$ (x-axis).

(d) [5 pts] Describe how these two plots differ. What are some of the possible reasons behind these differences? Pay particular attention to the shape of the dwarf, giant and supergiant sequences.

(e) [5 pts OPTIONAL] In general, what type of stars are the nearest stars? What type of stars are the brightest stars? Are there more cool/faint stars or more hot/bright stars in the vicinity around the Sun?

(4) A simple star model [20 pts]

Consider a star with mass $M$, radius $R$ and a density that declines with radius:

$$\rho(r) \propto 1 - \frac{r}{R}$$
(a) [15 pts] Using the equations of hydrostatic equilibrium and mass continuity, compute the pressure as a function of radius and show that the core pressure of this star can be written in the form

\[ P_c = kG \frac{M^2}{R^4} \]

Find the value of the numerical constant \( k \).

(b) [5 pts] Using the solar values for \( M \) and \( R \), determine numerical values for the core density and pressure, as well as the core temperature, \( T_c \), assuming an ideal, fully ionized gas with \( X = 0.33 \), \( Y = 0.46 \), \( Z = 0.21 \) (Note: this is different than the atmosphere!). Compare to the estimated core temperature of the Sun of \( 1.5 \times 10^7 \) K (see http://www.ap.stmarys.ca/~guenther/evolution/ssm1998.html). Note: you can assume the mean molecular weight for metals is \( \approx 2 \).

(5) Install MESA [20 pts]

This quarter we’re going to be using the MESA package to do full modeling of stellar interiors. This week your task is to download the package and run some of the example functions. First visit:

http://mesa.sourceforge.net/prereqs.html

and follow the instructions for installing pre-reqs (MESA SDK) and the code. There is a “troubleshooting” link if you are having problems. Make sure to start this early as (1) it is a large download and (2) there are often problems in installing the code on some operating systems.

Then run the pre-main sequence and post-main sequence examples described here:

http://mesa.sourceforge.net/starting.html

You should turn in screenshots of the completed HR diagrams and temperature versus density (TRho) plots that are produced by the program.