Outline for Lecture 19:

Announcements:
- Paper draft was due this morning – plan is to get you feedback by Friday
- Presentation draft on Friday – not mandatory, but advised (will get you feedback)

Review: stages of post-MS stellar evolution, He collapse threshold

He main sequence follows – shorter time scale
Estimate: \( t = \frac{[\text{energy/mass}] \times [\text{mass}]}{[\text{power}]} \)
\( L = 100 \text{ L}_\odot \) \( \leftarrow \) set by the temperature of the core and He energy generation rate
Energy/mass = 7.23 MeV / 3 He atoms = 7.23 MeV / 12 u = 1.2e-5 erg x NA /12 = 6x10^{17} \text{ erg/g} 
Cf. H fusion = 26.7 MeV / 4 H atoms = 4.3e-5 erg x Na/4 = 6.5x10^{18} \text{ erg/g} 
Let’s assume whole core fuses = 10% of M – same as H fusion 
\( t = t_\odot \times 0.1/100 = 10^{-3} t_\odot \approx 100 \text{ Myr} \)

after this double-shell burning AGB stars -> repeat of RGB with thermal pulsation 

enough to throw off atmosphere – example of Betelgeuse 
give rise to light echos – V838 Mon

Next stage set by mass of star – different outcomes

Low mass – atmosphere collapse & rebound – PN, modestly heavy elements (esp. organics)
Some properties of white dwarfs – example of Sirius B, spectra, density of WDs (4 among 50 closest stars, 2 are companions Sirius B and Procyon B)
WD luminosity fxn – sets the age (\( \sim \)9-11 Gyr), star formation history of Galaxy

Chandrasekhar mass for C/O (degenerate) white dwarf – note that \( \mu e = 2 \) gives MCh = 1.4 Msun 
Stars already more massive than this: next stages of element fusion
We are discussing post-MS evolution of a low-mass star like the Sun.

AGB stars represent last gasps of stellar burning, accompanied with massive pulsation, from shell burning, mass loss, and eventually formation of planetary nebula.
A0B stars undergo tremendous mass loss: \( \dot{M} \approx 10^{-4} M_\odot \text{yr}^{-1} \) (episodic)

Composition is enriched due to dredge-up & material addition.

Shells & dust + go, form cold enough to form grains + dust + produces a planetary nebulae.

\[ \text{GOES TO FORM ISM, NEXT GENERATION OF STARS} \]

What's next?

\( M < 4 M_\odot \Rightarrow \text{C/O core remains as envelope dissipates} \)

\( \Rightarrow \text{exposed core (very hot!) goes on to be a white dwarf} \)

\( 4 < M < 8 M_\odot \Rightarrow \text{add'l nucleosynthesis can occur} \)

\[ \begin{align*}
12^6 C + \frac{1}{2} 12^6 C & \rightarrow 16^8 O + 2^4 He & \Delta \varepsilon \text{ Endothermic} \\
( T = 6 \times 10^{8} \text{K}) & & \\
16^8 O + 16^8 O & \rightarrow 28^{16} Si + 2^4 He & \Delta \varepsilon \\
( T > 10^{9} \text{K}) & & \\
& & \\
\end{align*} \]

and on to

\( 56^{56} Fe - \text{peak of nuclear binding energy} \)
$M > 8 M_\odot$

Accretion:

\[ ^{14} \text{Si} \rightarrow ^{32} \text{S} \]

\[ ^{32} \text{S} \rightarrow ^{36} \text{Ar} \]

\[ ^{16} \text{S} \rightarrow ^{18} \text{Ar} \]

$3 \times 10^9 \text{ K} \rightarrow ^{52} \text{Cr} \rightarrow ^{56} \text{Ni} \leftarrow \text{past Fe peak} \rightarrow \text{end of nucleosynthesis}.$

Eventually core collapse $\Rightarrow$ SN (Type II)

Why collapse? Fe photodisintegrate:

\[ ^{56} \text{Fe} + \gamma \rightarrow ^{13} \text{He} + 4 \text{He} \]

\[ ^{4} \text{He} + \gamma \rightarrow 2^1 \text{p} + 2^0 \text{n} \]

\[ \Uparrow \]

Very high energy photons

\[ T > 10^9 \text{ K} \]

Loss of Fe + $\gamma$ degeneracy

$\Rightarrow$ loss of pressure support

How much energy released?

10 $M_\odot$ star $\rightarrow$ 1.3 $M_\odot$ core for Fe photodisintegration to start

\[ \Uparrow \]

\[ \rho \sim 10^{13} \text{ kg/m}^3 \Rightarrow R = \left( \frac{3}{4\pi \rho} \right)^{1/3} \approx 4 \times 10^5 \text{ m} \approx 0.1 \text{ R}_\odot \]

\[ \Uparrow \]

\[ \text{Energetic potential} \sim -\frac{3}{5} \frac{6M^2}{R} \approx -7 \times 10^{44} \text{ J} \]

Collapse to a 50km ball:

\[ U \sim -\frac{3}{5} \frac{6M^2}{R_{50}} \approx -10^{46} \text{ J} \approx -10^{46} \text{ J} \]

About right energy scale for SN explosion.
Why do AGB stars lose so much mass?

\[ T = 3 \times 10^4 \, \text{K} \]

E.g., Betelgeuse \[ M = 18 \, \text{M}_\odot \approx 3.6 \times 10^{31} \, \text{kg} \]

\[ R \approx 1200 \, \text{R}_\odot \approx 8.4 \times 10^9 \, \text{m} \]

\[ <\rho> = \frac{3}{4\pi} \frac{M}{R^3} \approx 10^{-5} \, \text{kg/m}^3 \approx 10^{-8} \, \text{g/cm}^3 \]

(\text{cf. air } \approx 1 \, \text{kg/m}^3)

\[ g = \frac{GM}{R^2} \approx 3.4 \times 10^{-3} \, \text{m/s}^2 \] (\text{cf. Earth } \approx 10^{-7} \, \text{m/s}^2)

\[ V_{\text{esc}} = \sqrt{2gR} \approx 80 \, \text{km/s} \] (\text{cf. Earth } \approx 11 \, \text{km/s})

\[ \frac{<v^2>}{<v^2> + \frac{3kT}{M_{\text{H}_2}}} \approx 1.2 \times 10^{-2} \approx \sqrt{\frac{<v^2>}{<v^2>}} \approx 350 \, \text{km/s} \]

\[ \nabla \]

thermal motion is enough!
The deaths of stars

We've traced the evolution of a star off the MS, after its exhaustion (AGB) (H-type) (H-TPP)

its core H → shell burning → core collapse → the D-shell → the burning →

the exhaustion + double shell burning → thermal pulses. The ultimate fate
now depends on the mass of the C/O core/canoe core.

Assume that the core can be treated as a separate object, i.e., with
negligible external pressure. The pressure can be written as:

\[
P = P_e + P_{\text{ion}} - \frac{\rho \mu T}{m_e m_n} + K \left( \frac{\rho}{m_e} \right)^\gamma + \frac{\rho \mu}{m_e m_n}
\]

\[\frac{1}{\mu_e} \frac{\rho}{\mu} \approx 0.5 \quad \frac{1}{\mu_e} = \frac{1}{\mu} \approx \frac{1}{2} \ll \frac{1}{\mu_e}
\]

degeneracy pressure, \( \gamma = \frac{5}{3} \) non-relativistic \( \rho \ll 10^6 \text{g/cm}^3 \)

critical \( \rho \sim 10^{13} \text{g/cm}^3 \)

The core pressure from hydrostatic equilibrium is:

\[
P_0 = \frac{G M C_e \rho_e}{R_e} \propto \text{central core density}
\]

where \( f \) is a constant \( \propto \rho / \rho_0 \)

\[
\Rightarrow f G M C_e \rho_0 = \frac{P_0 \mu T_0}{m_e m_n} + K \left( \frac{\rho_0}{m_e} \right)^\gamma
\]

\[
\Rightarrow \frac{K}{m_e m_n} T_0 = f G M C_e \rho_0^{4/3} - K\rho_0 m_e^{-4/3}
\]
For low density \( \gamma = 5/3 \), \( P_{\text{deq}} = K_1 \left( \frac{\rho}{M_c} \right)^{5/3} \)

\[
\mu_{\text{enuTo}} = \rho_0^{1/3} \left( f G M_c^{4/3} - K_1 \rho_0 M_c^{1/3} \right)
\]

\(\Rightarrow\) For low \( \rho_0 \), \( T_0 \propto \rho_0^{1/3} \) until a maximum is reached when

\[
\frac{dT_0}{d\rho_0} = 0 \Rightarrow \rho_{\text{max}} \left( \frac{f G}{2K_1} \right) M_c \mu_c^2
\]

\( T_{\text{to max}} = \frac{1}{2} \mu_c f G \left( \frac{1}{k} \right) M_c \mu_c^2 \)

\(\Rightarrow\) As \( \rho_0 \) increases, \( T_0 \) decreases and is zero when

\[
\rho_0 = \left( \frac{f G}{k_1} \right) M_c \mu_c^2 \Rightarrow 8 \rho_{\text{max}} \mu_c f G
\]

However, as \( \rho \) increase \( \gamma = 5/3 \rightarrow 4/3 + \varepsilon \) with \( \varepsilon \rightarrow 0 \)

\(\Rightarrow\) \( P_{\text{deq}} = K_2 \left( \frac{\rho}{M_c} \right)^{4/3 + \varepsilon} \)

Then

\[
\mu_{\text{enuTo}} = \rho_0^{1/3} \left( f G M_c^{4/3} - K_2 \rho_0 M_c^{-4/3} \right)
\]

in this case \( T_0 \) won't go to zero but will keep increasing as \( \rho_0^{1/3} \) in the values of the parentheses is \( \gamma > 0 \), i.e. for \( \varepsilon \rightarrow 0 \)

\[
M_c > \left( \frac{\mu_c}{f G} \right)^{3/2} \left( \frac{M_c}{M_{\text{ch}}} \right)^{2} = \frac{5.8 \mu_c f G}{M_{\text{ch}}}
\]

For \( M_c < M_{\text{ch}} \), we get a maximum temperature and then decreases to \( \varnothing \) at full support by degeneracy pressure \( w_1 \)

\[
\frac{P_{\text{max}}}{M_c} = \frac{1}{8} \left( \frac{K_2}{k_1} \right)^{3/4} \left( \frac{M_c}{M_{\text{ch}}} \right)^{2} = 2 \times 10^5 \text{D/cm}^3 \left( \frac{M_c}{M_{\text{ch}}} \right)^{2}
\]
and \( T_0 < T_{0,\text{ejecta}} = \frac{1}{4} \frac{M_0}{u_1} \left( \frac{M_e}{M_{\text{Ch}}_0} \right)^{3/4} \)

\[ = 5 \times 10^{36} \left( \frac{M_e}{M_{\text{Ch}}_0} \right)^{3/4} \]

In this case the core becomes a degenerate, supported object too cool to start the next phases of nuclear burning \( \rightarrow \) C/O white dwarf. The envelope falls in and rebounds, or blows out via stellar winds, oscillations, creating a planetary nebula. The core evolves and cools over time, with a mass-radius relationship set by non-relativistic degeneracy relation \( (n = 3/2 \text{ polytrope}) \ A \propto M^{-1/3} \)

Note that mass flow onto degenerate core can increase \( M > M_{\text{Ch}}_0 \), which can be the case for a giant/low mass companion overflowing its Roche lobe (cataclysmic variables). The result is a heating + detonation of C in shell regime that will effectively tear star apart - this is a Type Ia SN.

A core initially, with \( M_e < M_{\text{Ch}}_0 \) can also continue to grow from shell burning before envelope is lost. To con then grow and C flash will occur (core is degenerate). Core heats up + then expands. Convection may not be sufficient to carry away
the energy, so a shock wave can form the propogates outward at the local speed of sound. The compression + heating might ignite new nuclear burns - this is called a detonation front - or produce burns in its wake - a deflagration front. In these cases of non-hydrostatic burning, the entire carbon core can be burned; e.g. via \( \frac{12}{6} C + \frac{6}{6} C \rightarrow \frac{23}{11} N + \frac{1}{1} p \) \( \Delta E_0 = 2.2 \text{MeV} \)

\[ 20 \rightarrow 10 \text{Ne} + \frac{4}{2} x \] \( \Delta E_0 = 4.6 \text{MeV} \)

etc... etc...

include other chains resulting from \( p, \alpha \), etc., \( \Delta E_0 \approx 1.3 \text{MeV/amu} \)

\[ \frac{E}{m^2} = 1.3 \text{MeV} \cdot 1.6 \times 10^{-6} \text{erg/meV} \]

\[ = \frac{24 \text{amu} \times 1.67 	imes 10^{-24} \text{g/amu}}{24 \text{amu} \times 1.67 	imes 10^{-24} \text{g/amu}} \]

\( E = 2 \times 10^{51} \text{erg} \)

\( M_e \approx 1.5 M_0 \)

\( E \approx 2 \times 10^{51} \text{erg} \)

the binding energy of the same core is:

\[ E_B = -\frac{3}{2} \frac{G M_0^2}{R} \]

\[ \approx 1.5 \times 6.7 \times 10^{-8} \text{cm}^3/\text{g} \left( 1.5 \times 2 \times 10^{53} \text{g} \right)^2 \]

\[ = 2 \times 10^{51} \text{erg} \]

\[ R_{\text{core}} \approx \frac{1}{1.5} \text{g/cm}^3 \]

\[ \approx 5 \times 10^6 \text{cm} \]

\[ R_{\text{core}} = R_{\text{core}} \left( \frac{M}{M_{\text{core}}} \right) \]

\[ \approx 5 \times 10^8 \text{cm} \]

\[ \approx 1 \text{M}_e \]

Again this is a Type I SN