Lecture 3: Stellar magnitudes & the HR Diagram
3 October 2013

PRELIMS
• Announcements [5 min]
• APOD [5 min]
• Review [5 min]

MATERIAL [70 min]
• [20 min] Astrometry: proper motion & kinematics
• [35 min] Stellar brightnesses: Luminosity, flux, magnitudes

DEMONSTRATIONS/EXERCISES
• Calculate typical proper motion

MATERIALS
• Star chart from lab
• Burning questions box
Astrometry: Proper Motion & Stellar Kinematics

• Parallax – error fix (redo drawing, \(\tan \alpha \approx \alpha = 1 \text{ AU/1 pc}\))
• Proper motion: Stars aren’t actually “fixed in the firmament” – they move on small scales
  o Decomposition of velocity vector
    ▪ in “plane of sky” = proper motion)
    ▪ perpendicular to sky = radial velocity)
    ▪ Position angle – orientation of \(\mu\), going E of N
  o **CALCULATE:** Sun goes around Galaxy at 220 km/s; assuming most stars near us have similar orbits, estimate typical relative stellar velocities are of order 20 km/s. Compute the angular motion of a star at 10 pc moving at that speed perpendicular to our line of sight [60 sec]
    ▪ Answer: \(\alpha = s/d \Rightarrow d\alpha/dt = \mu = v_{\tan}/d = 2 \text{ km/s/pc x (1 pc/3x10^{13} \text{ km}) x (180^\circ/\pi \text{ radian}) x (3600 ''/^\circ) x (\pi x 10^7 \text{ s/yr}) = 0.4''/yr}\)
  o Highest \(\mu\) star is Barnard’s star: 10”/yr
  o Typical nearby \(\mu\) is much too small to “see” with the eye – even monitoring for several years!
  o Wasn’t until Halley in 1718 noticed a change in the position of a few stars compared to Hipparcos (1850 yr baseline!)
Today we can measure motions of $m''$/yr = 5 km/s/kpc; GAIA will measure down to 10 $\mu''$/yr = 50 km/s/Mpc

- Other motions on sky
  - binary orbit
  - other gravitational encounter
  - gravitational lensing (apparent astrometric perturbation – 1919 GR experiment)

- Kinematics
  - combine coordinates, parallax, proper motion and radial velocity and you have 6D position/velocity vector of a star
  - important for Galactic stellar kinematics: traces star clusters (co-moving stars) and gravitational potential of Galaxy (e.g., velocity curve)
  - Cartesian galactocentric coordinate system has XYZ and UVW tied to Galaxy
  - Show some fun orbits

QUESTIONS?

- Brightness scales of stars
  - Energy quantities:
    - LUMINOSITY – total power – constant
    - FLUX – L/A; spherical decreases as $1/r^2$ (derive)
    - Flux density – Flux/frequency = spectrum
- Intensity = Flux/frequency/sterradian = actual energy emitted from a parcel of gas

○ Magnitudes

- Magnitude scale – based on assumption we perceive logarithmically (e.g. decibels) – popular in 1850s, but in fact our perception is more like a power-law (Steven’s Law)
  - Hearing: $\psi \sim I^{0.67}$
  - Starlight: $\psi \sim I^{0.5}$
  - $\psi$ is the “psychophysical function” (magnitude of sensation)

- Based on Hipparchus ordering, we define star brightness in base 10 logarithmic magnitudes: $m - m_{\text{ref}} = -2.5 \log (f/f_{\text{ref}})$
  - $f$ is usually flux, but could be luminosity, flux density, intensity
  - Relative scale – requires a reference, commonly the bright A0 star Vega defined to have $m = 0$ at all wavelengths (but not $f = 0$!) => $m = -2.5 \log (f/f_{\text{Vega}})$
  - negative scaling – brighter things have lower numbers (even negative!)
  - every 2.5 magnitudes = 10x change in brightness

- Bolometric magnitudes ($m_{\text{bol}}$) assume total light at all wavelengths

- More often, magnitudes are given for specific wavelength regions defined by
standard filters – e.g., U (UV, 365 nm) B (blue, 445 nm) and V (visible, 551 nm)
  • Visible stars typically have V < 4-6 depending on site
  ▪ Color – subtract to filter magnitudes => ratio of flux
    • B-V < 0 => ratio of flux at 445 nm and 551 nm greater than 1? NO: greater than that of Vega!
  ▪ Bolometric correction: BC = m_{bol} – m

  o Apparent vs Absolute magnitudes => distance modulus
    ▪ Flux depends on luminosity & distance
    ▪ Choose a reference distance of 10 pc; derive
      M = m – 5 \log (d/10)
    ▪ M is a fundamental property of a star – intrinsic brightness
    ▪ m-M = 5 \log (d/10) => distance modulus
      • < 0 for stars > 10 pc => stars are intrinsically brighter than they appear if they are distant!
    ▪ Sun: M_{V} = 4.83, V = -26.7 (derive this yourself!)
Announcements

• HW #1 due TOMORROW @ 5pm in box outside my office SERF 340
• HW #2 now online; due Friday 11/11 @ 5pm
• Office hours tomorrow 11a-12p in SERF 340
• Burning questions?
• Grade ID
• “Our Star Will Die Alone” Fr & Sa @ 10:30pm on Galbraith south patio; free preview Th @ 8pm 10:30pm
Astronomy Picture of the Day 9/28/2013
http://www.star.ucl.ac.uk/~apod/apod/ap130928.html
Review

• Why is the parsec the natural unit of distance in stellar astrophysics? [2 reasons]
  – the nearest star (other than the Sun) is ~1 pc away
  – parallax measurement: 1 pc = 1 AU/1”

• What is the approximate mass density of stars in the neighborhood of the Sun?
  – $\approx 1 \, M_\odot / (4\pi/3 \times 1 \, pc^3) \approx 0.25 \, M_\odot/pc^3$ (really $0.08 \, M_\odot/pc^3$)

• What is the approximate radius of our Galaxy (starlight)?
  – 10 kpc – dark matter “halo” extends much further

• What is the current sidereal time?
  – $\approx 14h45m$ because sidereal time at midnight $\approx 0h$

• Point to the vernal equinox

• When might a scientist use ecliptic coordinates?
  – to map objects in the Solar System
Stellar Parallax

Apparent position of nearby star appears to move over course of year due to parallax effect.

$$\tan \pi \approx \pi \approx \frac{r}{D} \Rightarrow D \approx \frac{r}{\pi}$$

Typical parallaxes are < 1" \(\approx \frac{1}{3600} \approx 0.0000048\) rad.

- **\(r = 1\text{AU} = 1.5 \times 10^{11}\) m**
- **\(\pi (") = 0.0000048\) rad**

\[\Rightarrow D = \frac{3.086 \times 10^{16}\text{ m}}{\pi (")}\]

\[\text{or } D(\text{pc}) = \frac{1}{\pi (")}\]

Example (in 1838): Bessel measured 61 Cygni: \(\pi = 0.316\)

\[\Rightarrow D = 3.16\text{ pc} = 10.3\text{ ly}\]

(\text{true value} = 3.48\text{ pc})

1990 Hipparcos: accuracy \(\pm 1\) mas \(\Rightarrow\) distances to 1000pc = 1 kpc

2012 GAIA: accuracy \(\pm 20\) mas \(\Rightarrow\) 50 kpc (distance to Large Magellanic Cloud)

Can also turn around:

\[r(\text{AU}) = \Delta \theta (") D(\text{pc})\]

\(r(\text{AU}) = 10\) AU

Binary star at 20 pc.
Stellar Astrometry - proper motion

Velocity of a star can be decomposed into motion in plane of sky + along line of sight

\[ \mathbf{v} = (v_r, v_\theta) \]

\[ = (v_r, d\mu/d\ell) \]

\[ \text{distance} \quad \text{proper motion} \quad (2 \text{ components}) \]

\[ \mu = (\mu_r \cos \delta, \mu_\delta) \]

\[ \mu_r \approx \frac{\text{projected stellar proper motion}}{\ell} \]

\[ \phi = \text{position angle} = \tan^{-1} \frac{\mu_r \cos \delta}{\mu_\delta} \]

(measures degrees E from N)

Typical values are "/year or mas/yr" or mas/century

\[ |V_e| = V_{\tan} = \text{tangential velocity} \]

\[ V_{\tan} (\text{km/s}) = 4.74 \mu (\text{"/yr}) \times 10^3 (\text{pc}) \]

\[ |V| = V_{\text{tot}} = \text{total velocity} = \sqrt{V_r^2 + V_{\tan}^2} \]

Space velocities, commonly written in Galactic coordinates U,V,W

\[ W = \text{velocity toward NGP} \]

\[ V = \text{direction & galactic disk rotation at Sun} \]

\[ U = \text{toward (RHR) or away (LHR) center} \]

Note: \( \hat{z} \) of galaxy points toward southern galactic pole!
Stellar fluxes + magnitudes

We study stars primarily by the luminous energy they emit and we detect by our eyes, film, digital detectors, etc.

Poynting vector \( \mathbf{S} = \frac{1}{4\pi} \mathbf{E} \times \mathbf{B} \) \( \propto \text{units: W/m}^2 \) or flux

Assume star emits some amount of energy/time

\[ \text{Luminosity} = L \]

\[ \text{Flux at surface} \; 1^2 = \frac{L}{4\pi r_1^2} = \frac{L}{4\pi r_1^2} = F_1 \]

\[ \text{Flux at surface} \; 2^2 = \frac{L}{4\pi r_2^2} = F_2 \]

\[ \Rightarrow \frac{F_1}{F_2} = \left( \frac{r_2}{r_1} \right)^2 \]

\[ \text{Statement of energy conservation} \]

if \( r_1 \) = surface of star, then

\[ \text{Einstein equation} = \frac{L}{4\pi r_1^2} \]

\[ \text{Statement of energy conservation} \]

Note stars do not have "surfaces"; just level in atmosphere that photons stream freely (more about this later)

Energy detected by telescope \( = F \cdot \text{Area of detector} \cdot \text{integration} \)

E.g. Solar constant: \( F_0, \text{Earth} = \frac{L_0}{4\pi(1\text{AU})^2} \)

\[ = \frac{4 \times 10^{-26} \text{W}}{4\pi (1.5 \times 10^{11} \text{m})^2} \]

\[ \approx 1.6 \times 10^{-17} \text{W/m}^2 \approx 1400 \text{W/m}^2 \]

\[ \Rightarrow \text{Whole Earth} = \frac{L_0}{4\pi(1\text{AU})^2} \cdot \pi(R_{\text{Earth}})^2 \]

\[ = \frac{4 \times 10^{-26}}{4(1.5 \times 10^{11} \text{m})^2} (6 \times 10^6 \text{m})^2 \]

\[ \approx 1.6 \times 10^{17} \text{W} = 160 \text{PW} \]

World consumption \( \approx 15 \text{TW} \approx 0.01 \% \)
Magnitude system

Hipparchos (190-120 BCE) — devised first "brightness classification":
20 brightest stars = 1st magnitude; faintest visible star = 6th magnitude
logarithmic scale - senses are generally based as such (e.g. dB in sound)
5 mag = 100 x brightness \( \Rightarrow \) 1 mag = 100 \( \frac{1}{5} \) = 2.512 \( \times \) difference
definition is based on relative scale:

\[
m_1 - m_2 = -2.5 \log_{10} \left( \frac{F_1}{F_2} \right)
\]

or

\[
F_1 = F_2 \times 10^{-0.4(m_1 - m_2)}
\]

Vega system: commonly we choose a reference star to "zero" the scale,
Vega (α Lyrae) is the most common choice

\[
m = -2.5 \log_{10} \left( \frac{F}{F_{\text{Vega}}} \right)
\]

Note: Vega actually has \( V = 0.03 \), so slight offset in zero point
Vega is also a young star with circumstellar disk \( \Rightarrow \) redder
than it "should" be in the infrared

AB system: an increasingly popular system is the AB magnitude system

\[
m_{AB} = -2.5 \log_{10} \left( \frac{F}{10^{-0.60}} \right)
\]

\( F \) in erg/s/cm²/Å of flux/ frequency/sin

an object with constant flux density has zero color in AB system
common for extragalactic science, very deep surveys.
distance modulus: Because flux decreases as $1/d^2$, the apparent brightness can be compared to an absolute brightness to infer distance. Again a reference is needed, chosen to be 10 pc. (Why not sun?)

$$M - m = -2.5 \log_{10} \left( \frac{\text{Flux at 10 pc}}{\text{Flux}} \right)$$

absolute magnitude $=-2.5 \log_{10} \left( \frac{L/4\pi(10)^2}{L/4\pi d^2} \right)$

$= -2.5 \log_{10} \left( \frac{(d/10)^2}{d^2} \right)$

$$M - m = -5 \log_{10} \left( \frac{d}{10} \right) \approx -5 \log_{10} d(\text{pc}) + 5$$

E.g. Sun $m = -26.8$ (very bright!)

$$d = 1 \text{ AU} = \frac{1}{2 \times 10^5 \text{ pc}} \quad \left( \text{this number is 1 radian in } \right)$$

$$M = m - 5 \log_{10} \left( \frac{1/2 \times 10^5}{1/10} \right)$$

$= -26.8 + 5 \log_{10} 2 + 5 \log_{10} (10^6)$

$= 4.7 \approx \text{ at 10 pc, Sun is visible to naked eye, but not particularly bright}$

relative luminosities

$$M_* - M_B = -2.5 \log_{10} \left( \frac{L_\odot/4\pi(10)^2}{L_\odot/4\pi (10^6)^2} \right) = -2.5 \log_{10} \frac{L_*}{L_\odot}$$

$= \log_{10} L_\odot \cdot 10^{-0.4(M-4.7)}$

E.g.$: M^2 = 0.58 \Rightarrow L_V = 10^{-0.4(0.58-4.7)} L_\odot \approx 44 L_\odot \text{ very bright!}$

ESO 439-26 (white dwarf) $M_V = 17.4 \Rightarrow L = 8 \times 10^{-5} L_\odot \text{ very faint!}$
Filters & Colors

Typically we measure light through a particular filter – a piece of glass (or other transparent material) that passes only a range of wavelengths.

- **U**: 3200 - 4000 Å, ultraviolet/violet
- **B**: 4000 - 5000 Å, blue
- **V**: 5000 - 6000 Å, visible/visual
- **R**: 6000 - 8000 Å, red
- **I**: 7500 - 10000 Å, infrared
- **J**: 1.25 - 1.32 μm, NIR
- **H**: 1.5 - 1.8 μm, NIR
- **K**: 2.0 - 2.5 μm, NIR

We represent colors as a ratio of flux between two filters.

\[
J - K = -2.5 \log_{10} \left( \frac{f_J}{f(Vega)_J} \right) + 2.5 \log_{10} \left( \frac{f_K}{f(Vega)_K} \right)
\]

We can also define a bolometric correction that allows one to convert a filter magnitude (portion of the total flux) into a bolometric magnitude (total flux).

\[
BC = M_{bol} - V = M_{bol} - V
\]

Note: \( M_{bol} = 4.74 \) for the Sun

\[
\Rightarrow M_{bol} = 4.74 - 2.5 \log_{10} \left( \frac{L}{L_\odot} \right)
\]

for any star.
Standard Photometric Filters

Relative Transmission

Wavelength (angstroms)

3000 4000 5000 6000 7000 8000 9000 10,000

U B V R I

Typical B Star

http://www.skyandtelescope.com/howto/basics/Stellar_Magnitude_System.html